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AUTHOR Bulcock, J. W.; Beebe, M. J.
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ABSTRACT

The "primacy of reading" hypothesis as the explanation for the high correlation between children's competencies in literacy and numeracy is rejected in this paper. Instead, a "primacy of speech perception" hypothesis is examined. It is suggested that analysis-by-synthesis speech recognition procedures are acquired in early childhood and that the logic of the procedure is common to both arithmetic problem solving and reading comprehension. The paper discusses research that found that all the covariance between reading and numeration for a sample of grade four children was accounted for by three decoding strategies and asserts that the research confirmed the reciprocal interrelationship between literacy and numeracy. The question of research method is addressed and a discussion is provided of a new single equation estimation procedure called normalization regression analysis that was used for both recursive and nonrecursive model estimation in preference to the more traditional ordinary least squares and two-stage least squares regression methods. (MKM)

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A RE-EXAMINATION OF THE RELATIONSHIP BETWEEN LITERACY AND NUMERACY

J. W. Bulcock
Memorial University of Newfoundland

and

M. J. Beebe
University of Alberta

February 1980

(* J. W. Bulcock, Principal Investigator)

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ABSTRACT

The "primacy of reading" hypothesis as the explanation of the high correlation between children's competencies in literacy and numeracy is rejected. Instead a "primacy of speech perception" hypothesis is examined. Analysis-by-synthesis speech recognition procedures are acquired in early childhood prior to the development of basic literacy and numeracy. Yet, the logic of the analysis-by-synthesis procedure is common to both arithmetic problem solving and reading comprehension. In structural terms, competencies in all three decoding strategies {reading, numeration and speech recognition} are dependent on the child's ability to intuitively use phonetic, syntactic, and semantic rule-following or cueing strategies. Thus, it was found that all the covariance between reading and numeration for a sample of grade four children was accounted for by these strategies as mediated by the school dependent basic language skills. Similarly, the research confirmed the reciprocal interrelationship between literacy and numeracy. An elasticities analysis demonstrated, however, that reading and numeration, though highly responsive to one another, were not particularly responsive to changes in children's cueing strategies or basic skills performances. Some suggestions for further research were made in the light of this unanticipated finding.

The paper also addressed the problem of multicollinearity associated with ill-conditioned data sets. A new single equation estimation procedure called normalization regression analysis was used for both recursive and nonrecursive model estimation in preference to the more traditional ordinary least squares and two-stage least squares regression methods. Normalization regression analysis reduces, but does not eliminate, the harmful effects of collinearity. The several advantages of the new estimation procedure are described in the technical appendix.

J. W. Bulcock
M. J. Beebe
Edmonton, Alberta
February 1980

SUMMARY

The traditional explanation for the high correlation between children's competencies in literacy and numeracy emphasizes the primacy of reading. Underlying the explanation is the assumption that because there is a reading component to arithmetic problem solving, then reading is common to both. Thus, reading can be expected to influence the development of numeration more than numeration can be expected to influence reading.

As a consequence of this explanation much research into the nature of children's problem solving behaviour emphasized reading as a major factor predictive of verbal arithmetic problem solving. Recent reviews of this literature, however, show that the actual evidence to support the primacy of reading hypothesis is scanty and inconclusive at best. Effectively, this literature shows that gains in reading are associated with concomitant gains in arithmetic performance; but the usual conclusion that reading deficiencies hinder abstract thinking as represented by problem solving ability may not be legitimate because the conclusion calls for an examination of relationships of the "X affects Y, but Y may also affect X" variety, which are seldom specified or estimated.

There may, in fact, be common factors determining the covariation between literacy and numeracy as co-extensive outcomes of elementary schooling. This is what Australian researchers have recently suggested on the basis of their findings from the Australian national survey of literacy and numeracy. Thus, most Australian ten year-olds who failed to achieve mastery in reading did not necessarily fail to achieve mastery in numeration: a finding which counters the primacy of reading explanation, because according to this thesis most of those who fail to achieve reading competency would fail to achieve numeration competency.

Given such contradictions the primacy of reading thesis seems no longer tenable; to take its place a primacy of speech perception theory was proposed. Oracy precedes literacy and numeracy, and oracy in turn is dependent on speech perception; thus, the comprehension of speech or receptive language precedes productive speech, and speech perception or recognition precedes learning to read. It is tentatively posited, then, that there are conditional laws of language development such that phonological-morphological developments precede the exponential semantic-syntactic developments in early childhood. And for purposes of the present research it was assumed that numeration or arithmetical problem solving skills and reading comprehension ability develop co-extensively; and that such development could not occur in the absence of the child's successful initiation into the logic of speech recognition. In other words, neither numeracy nor literacy competency is possible in the absence of the successful development of speech recognition.

Speech recognition performance is characterized by a mode of decoding receptive speech known as analysis-by-synthesis. A cross-section of the speech recognition process begins with an accoustic signal or string of signals from a speaker (for example, a sentence or similar stretch of sound) and ends with the confirmation or nonconfirmation of the signal. If the signal is confirmed the successful synthesis will be followed by a preliminary analysis of the next signal and so on. But the logic of the analysis-by-synthesis decoding procedure is common to both reading and numeration.

The central notion in the analysis-by-synthesis model of speech perception is that the decoding process involves the making of predictions and inferences

about the meaning of the discourse. This notion is also central to the interpretation of the arithmetical problem solving behaviour of children which involves regular patterns of analysis-by-synthesis; that is, deduction from hypotheses followed by further prediction or inferencing based on the new synthesis and so on.

In the reading comprehension literature, the theory that stresses the importance of predicting or hypothesis testing is known as the "psycholinguistic guessing game theory." The guessing game theory is directly analogous to the speech recognition analysis-by-synthesis model. As the reader begins to analyze print he selects the cues to attend to on the basis of what he thinks he will find in the message. He will try to associate meaning with them by comparing the selected print information with what he already knows and generalizing the result to fit the situation he is reading about. The synthesis occurs when the results of the new reading situation and the reader's background knowledge are brought together. He is then able to make another prediction of what will likely follow on the basis of the synthesizing and assimilation of the old and new information. Thus, if further text information confirms the synthesis the reader can make another prediction of what is likely to occur next and so on.

If the reader is on the inside of analysis-by-synthesis decoding he will be able to select the most appropriate cues for decoding the message. The cues he can attend to include the graphic, phonic, semantic and syntactic. Thus, the best readers will probably be the most efficient in the application of grapho-phonic and syntactic-semantic cueing strategies. Since the solving of arithmetical problems involves translating common language statements into arithmetical statements; and since, therefore, analysis-by-synthesis decoding procedures are involved in numeration performances as well as reading performances

then it is probably the grapho-phonic and syntactic-semantic cueing strategies that account for the covariation in both literacy and numeracy because they are the strategies common to both.

Teachers, however, do not leave the acquisition of reading and numeration skills to chance; rather, in the earliest grades they introduce a program which is optimally designed at each grade level to achieve these goals. That is, they progressively initiate their pupils into the basic skills which involves developing the child's meaning vocabulary, language usage skills, and work-study skills. Thus, the effects of the cueing strategies on literacy and numeracy are in practice mediated by the basic skills.

If the primacy of speech perception theory is correct; that is, if the analysis-by-synthesis logic developed in order to decode receptive speech is common to both numeration and reading; and if the common linguistic strategies are successfully mediated by the school-dependent basic skills, then it can be hypothesized: (1) that the basic skills will mediate the effects of cueing strategies on reading and numeration performances; (2) that by grade four when pupils have started the switch from learning to read to reading to learn, both reading and numeration performances will be more dependent on syntactic-semantic strategies than the grapho-phonic; (3) that the cueing strategies and basic skills will account for thecovariation between literacy and numeracy; and (4) that the relationships between reading and numeration will be reciprocal, not primarily recursive.

In almost every respect the results confirmed the new theory. A recursive covariance model estimated using both ordinary least squares regression and normalization regression analysis supported hypothesis number three. A

reciprocal effects model was estimated using both two-stage least squares (2SLS) and two-stage normalization regression (2SNR) analysis procedures. Because 2SNR is a collinearity-reduced procedure (that is, 2SNR is a biased, reduced-variance estimating technique in the same family as ridge regression techniques) it generated more acceptable parameter estimates for the feedback model than the more conventional (but collinearity prone) 2SLS procedure. The 2SNR estimates (but not the 2SLS estimates) supported hypothesis number four.

The total causal effects of the source variables in the reciprocal effects model were calculated in an indirect effects analysis. The results confirmed hypotheses one and two. An elasticities analysis showed, however, that the reading and numeration outcomes though highly responsive to each other were not particularly responsive to changes in either the cueing strategies or the basic skills. This was an unanticipated finding which suggests that reading and numeration might be less responsive to cueing strategies and basic skills as proxies for analysis-by-synthesis processes than what it is these proxies actually stand for; that is, the (unknown) processes themselves. If this ex-post-facto explanation is correctthen more research is called for; and especially research into the processes underlying reading and numeration which lie in the realm of speech perception. It is believed then that research into the processes of oral discourse may prove a potentially important source of knowledge about the nature of literacy and numeracy.

1. The Problem[1]

It is well known that school performances in literacy and numeracy in the elementary grades covary; that is, students who perform well in reading will probably perform well in arithmetic.[2] Perhaps less well known is the fact that the association between literacy and numeracy tends to strengthen as the pupil moves from the middle grades to the junior secondary school. The correlations between reading and arithmetical problem solving for ten year-olds hover around the 0.6 level, while those for fourteen year-olds are around 0.7. Thus, for ten year-olds about 36 percent of the variance in one skill is explained by variance in the other; while for fourteen year-olds almost half the variance may be shared jointly.

Traditionally, the literacy-numeracy relationship has not been regarded as problematic; what little concern has been expressed has been centered on the magnitude of the impact of language factors and reading performance on the solution of verbal arithmetic problems. Yet, in a recent review of the research on arithmetic problem solving Hollander (1978, p.68) concludes that information on this problem is scanty and inconclusive at best.[3] The explanation with the greatest currency seems to emphasize the primacy of reading; namely, that because there is a reading component to numeration tests, then reading performance is a common

factor to both literacy and numeracy. It logically follows from this explanation; first, that reading deficient children will be seriously handicapped in their efforts to solve verbal arithmetic problems; and, second, that because reading is assumed to be logically prior to computational accuracy in arithmetic, reading will be a better predictor of numeration than the other way round.

At the time of writing, a review of the research in this area has failed to yield a single example of a double outcome, literacy-numeracy model, specified in such a way that its functional or mathematical form is isomorphic to explanations of the "X affects Y, but Y may also affect X" variety.[4] Yet, estimates of models of this form are necessary in order to adequately test the traditional explanation of the strong association between literacy and numeracy.

In 1976-77 the Australian Council for Educational Research released a three volume report of the results of a national study of literacy and numeracy (Keeves et al., 1976; Bourke and Lewis, 1976; Bourke and Keeves, 1977). The results failed to support the traditional "primacy of reading" hypothesis. If this hypothesis were "true", one would predict that students failing to achieve the criterion of mastery in reading would probably fail to achieve the criterion of mastery in numeration. The prediction stems directly from the argument that because reading is common to both skills, and because reading is logically prior to the

comprehension of verbal arithmetic problems, then reading deficits would handicap numeration more than numeration deficits would handicap reading.

Such was not the case. In fact, for ten year-olds the reverse was found. Bourke (1977, p.105) reports that 3,182 ten year-olds from the national probability sample failed to achieve mastery in reading; and of these 1,790 or 56 percent actually achieved mastery in numeration compared to 1,392 or 44 percent who failed to achieve mastery in numeration.[5] In following-up this refutation of the "primacy of reading" hypothesis Bourke (1977) reexamined the reading-numeration relationship after partialling out the effects of verbal sub-tests; that is, after controlling for their effects. He found that "when sub-tests were considered ... the relationships between mastery of Reading and Numeration Tests were not dependent on a reading component of the Numeration Tests." (p. 106, emphasis added).

The purpose of this study is to reformulate and estimate a model of literacy-numeracy outcomes of elementary schooling; that is, to test a new theory. Few would deny the importance of the problem. The fact is that most of the goals of people everywhere require education as the chief means of reaching them. For example, the initiation of the young into the major modes of understanding characteristic of national cultures is a necessary condition for the acquisition

of self-respect and for successful participation in daily life. Such goals are highly dependent on the successful acquisition of functional literacy and numeracy.

Theoretical concern is centered on the common causes of the covariation of literacy and numeracy. In view of the refutation of the traditional model a new model is called for; one with the null hypothesis that the model's source variables will completely account for this covariation. Accurate model specification also calls for the test of a feedback effects hypothesis; namely, that the literacy-numeracy relationship is reciprocal. Thirdly, the extent of the responsiveness of literacy and numeracy to their presumed common causes will be examined on the grounds that the magnitude of such relationships will determine the policy implications of the model.

In introducing the problem it was shown that recent research results have undermined traditional explanations of the strong association between reading and numeration; and it was argued that the importance of the problem necessitated its reexamination. In this study three interdependent questions are addressed:

(i) What are the common causes of literacy and numeracy?

(ii) To what extent are performances in literacy and numeracy mutually reinforcing or reciprocal? and

(iii) To what extent are school performances in reading and numeration responsive to their identifiable common causes?

2. Theoretical Framework

Because literacy and numeracy performances covary it is readily assumed that the two skills have common determinants. It can also be assumed that both have common roots in children's oral language usage. Oral language usage, in turn, is dependent on speech perception. According to Gibson and Levin (1975, p. 137) "before he speaks the child is semantically wise." Thus, the comprehension of speech or receptive language precedes productive speech; and, similarly, the comprehension and production of speech precedes learning to read (cf. Siegel, 1978, p. 59). In the authors' view, it is logical to examine the encoding process of oral language usage as a source of early reading and numeration performance.

It is posited, then, that there are conditional laws of child development in the sense that under the given conditions and notwithstanding developmental overlap, phonological development will invariably precede the morphological; that both will invariably precede exponential semantic and syntactic development in childhood; and that semantic knowledge will precede the syntactic (Gibson and Levin, 1975, p. 146). Though the magnitude of these relationships remain unestimated, threshold hypotheses would probably account for the resource transformations from one stage of language development to the next. The conditions governing several of these transformations seem well understood (e.g., Bronfenbrenner, 1979; Bruner and

Garton, 1978; Gibson and Levin, 1975; McNeill, 1970; Slobin, 1971). Since at any given time in early childhood there is likely to be mutually reinforcing feedback relationships between the phonological-morphological and semantic-syntactic outcomes of language development the resultant system of learning behaviours could prove predictive, though not deterministic, causes of language development at a later point in time. The structure of such relationships would constitute a helix; though todate the precise structure is unknown.

Before examining the reading comprehension and numeration literature for common elements the sources of both processes in speech perception is investigated.

Speech Comprehension

A model of speech comprehension has been formulated by Halle and Stevens (1964, 1967) which is usually referred to as the analysis-by-synthesis model. A cross-section of the process begins with a message or signal from a speaker and ends with the listener's confirmation or nonconfirmation of the message. Confirmation of the signal is the cue triggering the anticipation of what the message will contain next. Because the linear flow is interrupted by the necessity of listeners verifying their "guesses" at meaning against the actual signals the process is both oscillatory and iterative, though iterations are only necessary when a signal is not confirmed and the listener has to "guess again." If the iterations do not converge in the sense that the expected meanings are not

verified the message will not have been understood.

If the cross-sectional process from signal to confirmation or nonconfirmation is called a fixation, some five steps can be identified in each fixation. In step one accoustic signals from the speaker are received by the listener including the speaker's articulatory characteristics, gestures, and the phonetic environment. In step two the listener selects or samples units or patterns of the signal for synthesis. Though little experimental evidence throws light on the listener's criteria of signal pattern selection it is hypothesized that selection is based on the listener's awareness of both the accoustic and nonaccoustic constraints governed by phonetic, semantic, and syntatic rules. Phonetic rules govern the sounds uttered, semantic rules govern the words spoken, and syntactic rules govern the word order. Collectively they constitute the listener's strategy for decoding the signals.

More specifically, these intuitive language rules constitute the cues which trigger the listener's prediction strategy in step three. This is the guessing, hypothesizing or predicting stage; which is not to say that a listener is free to make an infinite number of guesses. Rather, knowledge of the rules of the language provide the clues for constructing and testing the general hypotheses about the meaning of the discourse. The constructions must be open enough to promote confirmation or synthesis in step four through verification against the original signals in step two. In other words,

the listener's prediction is checked through a feedback procedure with the previously selected signal to "see" the actual approximation with the message. Confirmation constitutes readiness in step five to anticipate what is coming next. Lack of confirmation may generate a replication of step three in a new attempt to achieve confirmation.

In structural terms it is hypothesized that speech comprehension will be responsive to the listener's intuitive knowledge of the underlying phonetic, semantic, and syntactic language rules, plus their interaction with the listener's background of experience.[6]

Numerical Problem Solving

Consider the following algebraic word problem adapted from Paige and Simon (1966, p. 53):

> If the number of papers Tom delivers is twice the square of 20 percent of the number of homes and apartment dwellings in the neighbourhood, and the number of homes and apartment dwellings is 20, what is the number of papers Tom delivers?

The problem is edited in Table 1 to illustrate the complexity of the task of decoding or translating from common language statements into algebraic statements; that is, from one symbolic system to another.

Table 1 about here

The algebraic statements give equations of the form:

$$x_1 = 2 * (0.2 * x_2)^2$$

$$x_2 = 20$$

$$? = x_1$$

Table 1

An Edited Version of the Translation of a Verbal Algebraic Problem from Common Language Statements to Algebraic Statements

Common Language Statements	Semantic Elements	Syntactic Elements	Algebraic Statements	No. Units
The number of papers Tom delivers	Substantive meaning to be translated into variable name	---	x_1 (an unknown)	1
is	---	verb	= (arithmetic operator)	2
twice	mathematical meaning	---	two times, or 2* (* = times)	3
the square of	mathematical meaning	---	square $(...)^2$	4
20 percent	mathematical meaning	---	two tenths, or, 0.2	5
of	---	functional operator	times, or * (cf. unit #3)	6
the number of homes and apartment dwellings in the neighbourhood	substantive meaning to be translated into variable name	---	x_2 (another unknown)	7
, and	---	conjunction	indicates end of clause	8
the number of homes and apartment dwellings in the neighbourhood	same substantive meaning as unit #7 variable	---	see unit #7	9
is	---	verb	= (arithmetic operator)	10
20:	mathematical meaning	---	20	11
what:	---	functional operator	?	12
is:	---	verb	= (arithmetic operator)	13
the number of papers Tom delivers?	same substantive meaning as unit #1	---	see unit #1	14

Solving the equations for x_1 gives the following result: $x_1 = 2 * (0.2 * x_2)^2$; and since $x_2 = 20$, then

$$x_1 = 2 * (0.2 * 20)^2 = 32$$

The successive transformations of the problem from common language input to the specification of the formal algebraic equations involves several steps. First, the substitution of algebraic symbols for common language statements involves identification of meanings (semantics) and in this regard it is as important to know what meanings to ignore as to know what meanings are essential. The essential meanings are the mathematical ones; namely, "two times" for twice, "square" for square of, and "percent" for percent. Second, it is important to be able to respond appropriately to grammatical (syntax) clues; that is, to be able to distinguish between words which carry substantive meanings and those that have syntactic functions.

Thus, a distinction has to be made between words which carry meanings and words which have grammatical functions. Those that carry meanings are of two kinds; ones with arithmetical meanings and ones with substantive meanings which are not arithmetical. The problem solver must understand the mathematical convention that substantive terms or phrases such as "the number of papers Tom delivers" or "the number of homes ... in the neighbourhood" are to be translated into variable names. The conventional names are of the X_1, X_2 variety. The problem solver has also to be able to differentiate between phrases

which name the same variable and those that name different variables.

In terms of syntax there are operators (op.), verbs, pronouns, and delimiters (DLM), several of which are illustrated by the following example where the words are tagged by function:

> The number (of/op.) papers Tom (delivers/verb) is 2 (times/op.) the (square/op.) (of/op.) 20 (percent/op.) (of/op.) the number (of/op.) homes and apartment dwellings (in/op.) the neighbourhood and the number (of/op.) homes and apartment dwellings (is/verb) 20, (what/Q word) (is/verb) the number (of/op.) papers Tom (delivers/verb) (Q mark/DLM).

Different scholars have identified different stages in the problem-solving process. Bloom and Broder (1950) identify four components: understanding the nature of the problem, understanding the ideas contained in the problem, general approach to the solution of problems, and attitudes toward the solving of problems. Gagné (1966, p. 138) also identifies four stages: (i) statement of the problem; (ii) defining the problem by distinguishing essential features; (iii) searching for and formulating hypotheses; and (iv) verifying the solution. Gary and Kingsley (1970, p. 464) identify three steps: a search phase, a functional solution phase, and a verification phase. In recent years research in mathematical problem-solving has concentrated largely on the functional solution or hypothesis formulation stage (Bloom and Broder's stage 3, Gagné's stage 3, and Gary and Kingsley's stage 2) in an effort to identify the rule-following cues (rules of thumb or heuristics) used by problem-solvers (cf. Blake, 1976; Kantowski, 1977; Kilpatrick, 1967; Pereira-Mendoza, 1979).

Though none of these researchers suggests that heuristic strategies may be proxies for thinking processes rooted in language comprehension as shown by the analysis of an algebraic problem in Table 1, there is

growing consensus that the introduction of a goal oriented heuristic corresponds to insight, and that at that point a path to a computational solution is characterized by regular analysis-by-synthesis patterns. Kantowski (1977, p. 166), for example, notes that "evidence of regular patterns of analysis-and-synthesis (deduction inferred from hypotheses followed by synthesis, then by further inferences from the new synthesis and so forth) was noted in the solutions of problems with scores above the median."

Reading Comprehension

Consider the following passage by a grade two boy. The selection is the beginning of a story that the seven year-old read aloud to the researcher.

1. Walters
One day Bob Watson was sick.

He had to stay home from school. He

2. © started st.
stayed in bed all day. And he looked

3. © at 4. helping
out the window to see what was happening

in the street.

5. going
He saw Mrs. Miller go to the store.

6. Bark Bu
He saw Mr. Burke bring the mail. He

saw Mr. Goodman bring the milk.

7. © down
Then he saw something bad. He

8. © smock
saw dark smoke coming out of the window

of Mrs. Miller's house. 9. (c) Dob Bob knew that

10. (c) at once no one was in the house to see the smoke.

((c) = corrected)

Though the reader made ten mistakes or miscues while reading the three short paragraphs there is no doubt that he understood the message the author was trying to convey. Each mistake is a substitution miscue which is the most common miscue made by beginning readers. (cf. Beebe, 1976; Goodman, 1971; Goodman and Burke, 1968; Weber, 1970). Readers can either correct substitution miscues or continue reading without correcting. If the reader fails to correct there are two other possibilities. First, the substitution may be syntactically or semantically acceptable; that is, the substitution is both grammatically equivalent to the correct word, and also an accurate representation of the meaning. Second, the substitution miscue may be syntactically-semantically unacceptable for any one of three reasons: (i) the substitution is syntactically acceptable but semantically unacceptable; (ii) the substitution is syntactically unacceptable, and in rare instances, semantically acceptable; or (iii) the substitution is both syntactically and semantically unacceptable. These are known as non-acceptable miscues.

In the first paragraph of the example the first, fourth, and fifth miscues ("Walters", "helping", and "road") are noncorrected miscues, while the second and third ("started" and "at") are classified as corrections because they were subsequently corrected after the child read a word or two further into the text as indicated by the length of the underlined phrase. Of the three noncorrections (1, 4, and 5), miscues one and five are acceptable, while miscue four is a nonacceptable miscue because while it is syntactically

acceptable it is semantically unacceptable.

Comprehension in reading involves using a set of procedures that allows the reader to select from his personal experience and knowledge about the world, those concepts or ideas that are relevant to the text at hand. As a reader works through a passage, he often elaborates on what is implied by the author and arrives at intended inferences. The process is referred to as inferencing and is often described as "reading beyond or between the lines." But other processes are involved as well including predicting, hypothesizing or guessing.

Prediction is the basis of comprehension (Smith, 1978). Each activity we perform is dependent on our anticipations and expectations about that activity. Such expectations are a result of our knowledge and experience of the world; that is, we are in a constant state of anticipation about how each activity will "turn out" based on our past experience and our ability to generalize from past experience. In order for us to make sense of our activities, we must be able to assimilate the incoming information into what we already know; that is, to be able to fit it into our existing knowledge so that it is in line with our anticipated outcome of the activity.

If the outcome does not coincide with our expectations, an imbalance occurs between our existing knowledge and the incoming information. We must then adjust, or accommodate our known information to incorporate the new information, or if such modification is impossible (because of what we know), we must discard the new information as being false or inaccurate; hence, unacceptable. At this point it is often necessary to alter one's expectations for the activity and to repeat the activity so that a new, more acceptable interpretation or judgment about the results of the activity can be made.

Because reading is an activity and because we predict or hypothesize about the outcomes of every activity we engage in, every reader must be able to make use of questions or expectations about the material to be read if he is to understand the material. When prediction procedures, or hypothesis testing activities are used to explain what readers do as they read, the resulting theory is usually referred to as the psycholinguistic guessing game theory of reading (Goodman, 1970). It is the direct analogue of the analysis-by-synthesis model of speech recognition (Neisser, 1967; Gibson and Levin, 1975).

Basically the theory claims that a reader utilizes his knowledge of the world, his knowledge of language, and his knowledge of the reading task at hand to analyze the situation in which he finds himself. As he begins to analyze the print, he selects the cues to attend to. Once the appropriate cues are selected in accordance with his prediction of what he thinks he ought to find in the message, he tries to associate meaning with them by comparing the selected print information to what he already knows; generalizing the results to fit the new situation he is reading about; and synthesizing or bringing together the results of the new situation and his already existing knowledge. As he synthesizes the new ideas he assimilates or incorporates them into his knowledge domain and is then able to make another prediction of what will likely follow based on the synthesizing and assimilation of his old and new information.

If the reading activity takes place in a situation where his knowledge background is not adequate for the demands that the print will make, he will be unable to pose adequate questions or determine appropriate expectations (that is, to predict) and, hence, will be unable to comprehend. This means that successful synthesizing depends on the amount of the reader's prior

knowledge relevant to the topic.

Not only is a reader dependent on background knowledge, but also equally dependent on knowledge of the language and the meaning conveyed through language as was illustrated by the miscue analysis of the above passage. As a reader reads, he has certain expectations that what he reads will sound like meaningful language; that is, will be grammatically acceptable and meaningful. As he begins reading, the reader selects graphic (print) cues in accordance with these expectations and uses this information to formulate a "guess" or to make a prediction about the intent of the message.

If, when the reader tries to associate those cues with his knowledge of meaningful language, the comparison is favourable, he is able to synthesize the information and check it out with subsequent information in the text. If further text information confirms his synthesis, the expectation or prediction of what ought to be there, is confirmed and he can make a further prediction about what is likely to occur next. If the guess or prediction is not consistent with his notion of meaningful language as well as with the forthcoming text, the reader recognizes that he has misread at least some of the information and must, therefore, reread or reinspect the print in order to alter his choice of cues; this time based on a different expectation of what he will find.

It seems to follow then that the reader is constantly feeding back the information received from the cues selected in the print to see whether it conforms to his anticipatory hunches, guesses, or hypotheses. If the feedback is positive the next unit of print is ready for processing. If the feedback is negative -- in effect the message does not"fit" --the reader will have to go back to reread the preceding unit of print in order to obtain additional or alternate information. If this reading version of the analysis-by-synthesis model holds any validity, it implies that readers need to be encour-

aged to expect that what they read will sound like meaningful language to them and that it will be associated in some way with what they already know.

All this is not to say that the analysis-by-synthesis model has widespread acceptance in reading circles. The fact is that the theory has been criticized on several grounds, two of which are referred to. First, the theory has been found lacking in specificity and, therefore, according to Gibson and Levin (1975, p. 453) "the theory is too imprecise to account for all kinds of reading." For example, experienced teachers have found that beginning reading is probably best treated as a precise and unambiguous process; one that involves the detailed and sequential perception of letters, words, spelling patterns, phrases, and sentences.

This notion of the importance of precision in the early development of grapho-phonic skills is congruent with William Gray's (1956) first two stages of learning to read; namely, the readiness phase developed in kindergarten, and the initial stage which begins in grade one, though there is considerable variability between children. Emphasis in stage two usually includes the development of a sight vocabulary along with instruction designed to help children recognize letter-sound relationships, and the use of contextual clues in pronunciation and decoding meanings.

It is not until pupils gain mastery over the grapho-phonic relationships in the first two phases of learning to read that emphasis on reading as a precise process may be relaxed. Thus, it is not until children enter a a stage of rapid progress which Gray referred to as the development of fundamental reading skills, habits, attitudes and tastes that the principles of learning to read as derived from the analysis-by-synthesis model seem relevant.

Though it seems, then, as if this criticism of the analysis-by-synthesis model is valid, we are inclined to regard it nevertheless, as naive criticism since it lacks appreciation of the flexibility and self-correcting features of the incremental modelbuilding tradition. The criticism does not seem to recognize that models are not "proved", only falsified; that, therefore, theories are in a constant state of flux. Seemingly contradictory or competing notions are sometimes found to be complementary; and occasionally the generalization of a theory will incorporate previously discrete theoretical elements. Theories, then, are never fully-developed as this criticism implies, but rather under constant examination and assessment which results in their extension, modification and elaboration. Criticism of the kind that a theory is too imprecise need not mean that the theory is invalid or inappropriate; rather that it is in need of modification or further elaboration. We regard the analysis-by-sythesis model because of its generality a suitable starting point for model formulation, estimation and development.

In the second place the analysis-by-synthesis model has been criticized on the grounds that it lacks predictability. If a theory lacks predictability or if it cannot be formulated in a falsifiable manner, then the very authenticity of the theory is at stake. The analysis-by-synthesis model formulated by the Goodman-Goodman-Burke group has been criticized on these second grounds. (cf. Harker, 1973). This was probably a legitimate criticism at the time, but the results presented in this study are sufficient to refute the criticism -- though the estimated model is far from being deterministic. Recently, Beebe (1980) formulated and estimated the basic equations of the psycholinguistic model of reading. (Goodman and Burke, 1968; Goodman and Burke, 1969; and Y. Goodman, 1971). The results demonstrated its predictive qualities. Beebe

and Bulcock (1979) have also developed and estimated an extended and elaborated analysis-by-synthesis model of both reading comprehension and retelling. The model had both a good fit to the data and high predictability.

Thus, despite the criticism, the authors find ample evidence supportive of the propositions defining the analysis-by-synthesis (or psycholinguistic guessing game) model of reading comprehension. The logic of analysis-by-synthesis decoding which is first developed in early childhood as a means of decoding receptive speech seems as central to learning to read as it is to understanding speech.

The Transformation of Linguistic Resources

During the earliest stages of reading a child tends to be dependent on the grapho-phonic strategy. Emphasis is placed on learning to associate letters with sounds that the child can convert into meaning by incorporating his knowledge of languages and experiences into his reading. As the beginning reader becomes more proficient, grapho-phonic competency can be taken for granted, and at this point the reader can be encouraged to utilize his intrinsic knowledge of the language and his experiences of the world around him in an effort to decode increasingly complex passages. In other words, as the reader gains proficiency with letter-sound relationships, he can relax somewhat in this area and concentrate on more complex reading material which requires that greater attention be given to sentence structure (syntax) and complex concepts (semantics).

Above, it was shown that the logic of the reading process was parallell to that of speech comprehension. In other words, children who are proficient in listening and speaking will already possess the necessary cognitive resources for transforming learning to read skills into those required for reading to learn. It is instructive, however, to note how teachers facilitate this transformation of one set of resources into additional resources. School learning theorists such as Carroll (1963)

and Bloom (1976) argue that a student's achievement is a function of his prior learning behaviours and characteristics, and that their effects are both direct and indirect. Direct effects refer to the fact that a student's achievements at any point in time are a function of prior knowledge and attitudes (the entry behaviours), while indirect effects refer to the effects of entry behaviours which are mediated by exposure to what is learned (the opportunity to learn or time on task), and how it is taught (the quality of instruction).

Teachers in the early grades provide treatments designed to advance learning opportunities; for example, to develop mastery in literacy and numeracy. Study skills are developed through use of dictionaries, reference books and other library resources. From these sources students practice using reading aids: maps, graphs, tables, and illustrations. Concomitantly, the student's stock of concepts is enlarged, and since these increasingly relate to other content fields, reading and language instruction begin to interface with instruction in other subject areas. As reading interests broaden and as reading proficiency increases -- often exponentially -- the literature read will extend beyond the experience level of the student with the result that reading assumes a more vicarious nature (Gray, 1956). Basic skills approaches described by authors such as Russell (1968, chapters 9, 10 and 11), Tinker and McCullough (1975, chapters 7, 8, and 10) and Durkin (1978, chapters 8, 9, 11, 12, 14 and 17) formalize the resource conversion process. If the child's psycholinguistic resources are regarded as primary resources and literacy and numeracy as secondary resources, then the basic skills may be regarded as the mediating linkages or the institutionalized mechanisms for converting the primary into the secondary resources.

3. Models of Literacy and Numeracy

In theory, the roots of early competencies in reading and arithmetic problem solving are traceable to the development of speech perception. It was demonstrated in the previous section of the paper that the analysis-by-synthesis model commonly used to explain the comprehension of speech had analogous referents in both theories of numeration and reading comprehension; and that to this extent all three had common logical elements. These include: (i) the making of hypotheses (guesses or predictions) about the meaning of the communication or nature of the problem, followed by (ii) the application of language or numeration rules to deduce what the input would be like if the hypothesis were true, (iii) checking or verifying the input against the prior information, and (iv) the making of further inferences on the basis of this new synthesis. It was shown, then, that both reading and numeration behaviours were highly dependent on the student's analysis-by-synthesis strategies.

How does the observer know that the reader or problem-solver is on the inside of the necessary analysis-by-synthesis symbolic processing procedures? The answer would seem to be that it depends on how efficient the pupil is in applying the graphophonic, syntactic and semantic rules governing these behaviours; that is, linguistic hypothesizing. By giving pupils passages to read which are challenging enough that they will make mistakes the RMI can be used to measure the major linguistic variables. The processes themselves which underlie the rule-following behaviour cannot be observed, only inferred from the observation of children's mistakes in problem-solving or miscues in reading (cf. Platt, 1964). Thus, for example, the smaller the proportion of miscues lacking graphic similarity

or the smaller the proportion of unacceptable semantic miscues, the more efficient the respondent's application of graphic and semantic rule-following strategies. Because the methods of inquiry in mathematical problem-solving are usually of the introspective/retrospective variety there are problems related to the reliability and validity of measurement. Further, there is the problem of time. It takes several hours to analyze the protocol of a single subject (cf. Pereira-Mendoza, 1979). Because the RMI procedures are more consistent, though coding is still time-consuming, and because the linguistic strategies required for meaning seeking in reading are the same as those required for problem solving in arithmetic, the present research measures the linguistic logic of information processing using the techniques developed by reading researchers rather than those of the mathematical problem-solving researchers.

If the linguistic strategies are common to both reading and numeration behaviours then they will account for the covariation between literacy and numeracy. Teachers, however, do not leave the transformation of primary language resources into desirable secondary resources to chance. Rather, in the earliest grades a program of basic skills development is introduced to promote the necessary resource conversion. Thus models of reading and numeration would be inadequately specified if the basic skills were not included as vital mediating mechanisms.

The problem which was stated in the form of three questions in the introductory section of the ~~paper~~ report, calls for the formulation and estimation of four models. First, a prediction model of early reading comprehension. Second, a prediction model of early numeration performance. Third, a covariation model in which the common determinants of reading and numeracy

in models one and two are used to "break down" the covariation between both outcomes. Actually, it is not necessary to estimate these models separately because they can be integrated as shown in Figure 1 through the estimation of only three structural equations. Fourth, the second question calls for the estimation of a reciprocal effects model. Once the structural parameters of the fourth model have been estimated the coefficients can be used to calculate model elasticities in order to provide unambiguous answers to the third question. In each model, then, the source variables -- graphic, phonic, syntactic and semantic performances -- are mediated in their effects on reading comprehension and numeration by the basic skills -- vocabulary, language skills, and work study skills.

Our thesis should be clear; namely, that three modes of comprehending -- speech perception, reading, and numeration -- have a common logical basis in the fact that they each call for guessing, hypothesizing or predicting behaviours, which when used in concert with background knowledge facilitate the making of inferences, which takes the listener, reader, or problem-solver beyond the literal meaning of the communication in question. We agree with Smith (1978) that prediction is the basis of comprehension; that in order for us to make sense out of our communications and activities we have to assimilate the incoming information into what we already know so that it is congruent with our expectations and anticipated outcomes of the activity. Predicting, guessing or hypothesizing in this sense means posing questions that will reduce uncertainty about the activity or event; and when the questions are answered such that uncertainty is eliminated comprehension occurs. It is worth noting in passing that this thesis is similar to that of Leonard Meyer who uses the notions of anticipation

and expectations -- that is, prediction and the reduction of uncertainty -- to explain the aesthetic appreciation of formal music, or any other symbolic art form. What makes sound so enjoyable says Meyer (1956, p. 59) is that:

> ... we are in a sense, constantly expecting. Under certain conditions we expect change, under others continuity, and under still others, repetition; until finally we expect the conclusion of the piece. Thus, in a very general way expectation is always ahead of the music, creating a background of diffuse tension against which particular delays articulate the affective cause and create meaning.

The main hypotheses generated by these models are the following:

(i) that the basic skills will mediate the effects of intuitive linguistic rule-following strategies as predictors of reading comprehension and arithmetical problem-solving behaviours;

(ii) that both arithmetical problem-solving and reading comprehension at the grade four level will be more dependent on syntactic and semantic strategies than the graphic and phonic;

(iii) that the linguistic rule-following strategies and basic skills will account for the covariation between literacy and numeracy; and

(iv) that the relationships between reading and numeration will be reciprocal in that one will stimulate performance in the other.

4. Method

Sample

Ninety four grade four pupils enrolled in two urban schools constituted the case base. Since it was necessary for all the subjects to be able to read the same passage, yet make mistakes, only those who were able to comprehend at least seventy five percent of what they read on Maze 5 and

and Maze 6 of the Guthrie-Siefert Maze Task were selected. All, then, would be able to read a selected passage difficult enough to produce reading miscues. Seventy four percent of the grade four children were selected -- 46 boys and 48 girls.

Materials

The instrument used for classifying the miscues, hence, for measuring the graphic, phonic, syntax and semantic variables was the Reading Miscue Inventory (RMI) developed by Y. Goodman and Burke (1972).

The Guthrie-Siefert Maze Task is a modification of the cloze procedure. Instead of inserting a word in a blank, the reader must select one of three alternative words that were embedded for every fifth word in the story. The percentage of correct selections indicates the degree of comprehension of the passage. Each test (Maze 5 and Maze 6) contains 28 items. The cut-off for inclusion in the sample was 40 points out of 56, which meant that the sample students comprehended at least 70 percent of what they read. Guthrie (1973) describes the psychometric properties of the instrument and shows that it correlates at the 0.82 level with the Gates-MacGinitie Comprehension Subtest.

The reading comprehension and arithmetic problem solving measures were grade equivalent scores obtained from the administration of a battery of tests known as the Canadian Tests of Basic Skills (King et al., 1973). King et al. (1975) report a split half reliability for the Reading Comprehension Subtest of 0.92, and a split half reliability of 0.90 for the Arithmetic Subtest.

Measures for the basic skills were also obtained from the Canadian Tests of Basic Skills. Meaning and sight vocabulary was measured by a subtest in which the student was required to identify words by sight and then select the word which conveyed the appropriate meaning. The spelling, punctuation, and language usage subtests were aggregated to form a language skill variable. Map reading, reading graphs and tables, and knowledge and use of reference materials were three subtests which were aggregated to form a work-study skills variable. A single outward indicator latent variable called "basic skills" was constructed from these three aggregated variables which had an omega reliability of 0.854.

Procedures

Step one in the data gathering procedure involved oral reading of the selected passage ("Space Pet"), the retelling of the story by the subject, and the audio-taping of these procedures. The second step involved replaying the tape in order to mark each miscue. The week following completion of the audio-taping the local school authority administered the Canadian Tests of Basic Skills. The reading comprehension, arithmetic problem solving, vocabulary, language and work-study skills scores were merged with the RMI scores for the sample children.

Cueing Strategy Variables

The procedures for gathering and coding the miscue data were identical to those recommended in the RMI. The graphic miscue variable represents a proportionate measure of each respondent's miscues which lacked graphic similarity. The proportion was obtained by dividing the number of dissimilar responses by the total number of responses in the single-word or non-word substitution categories. Thus the graphic component is scored such that the higher the score the lower the graphic performance. This scoring

system preserves the miscue feature that makes it dissimilar from expected responses.

The same procedure was used in constructing the phonics miscue variable which is referred to as the proportion of miscues lacking phonic similarity.

Only syntactic weaknesses were used in calculating an individual's unacceptable syntactic miscues; and, similarly, only comprehension losses were used in calculating an individual's unacceptable semantic miscues. Both were converted into proportions by dividing the unaccepted miscues by the respondent's total number of miscues minus the number of corrections. Thus, the variables were called the proportion of unacceptable syntactic miscues and proportion of unacceptable semantic miscues respectively. The related hypotheses would read as follows: the greater the proportion of unacceptable miscues the lower the reading comprehension score and so on.

Two inward-indicator latent constructs were derived from the observed cueing strategy variables through application of factor analytic techniques using oblique rotation. Their omega reliabilities (Heise and Bohrnstedt, 1970, p. 115) were .918 and .723 respectively. See Figure 1 for the structure of the outer relationships of these variables and for the stucture of inner relationships between the latent constructs and the outcomes.

Estimating Procedures

Complex models of the kind specified in the previous section of the paper and illustrated in Figure 1 are frequently plagued by multicollinearity; that is, the condition where predictor vectors depart from orthogonality or from the condition of linear independence. It will be demonstrated

in the findings section that the present study is no exception. When this happens ordinary least squares (OLS) estimates are likely to be too large in magnitude, and too sensitive to sampling error and specification error. Sometimes OLS estimates will have the wrong sign, and usually under these conditions the significance of the multiple R square of the estimates will be reduced so that a good fit need not mean good forecasting power. Recently, a few researchers have begun to develop solutions to the problem. Two promising lines have emerged: (i) the use of models involving latent variables which include within themselves much of the multicollinearity, and (ii) the use of biased / reduced-variance estimators, such as ridge estimators.[7] Both procedures are used in the present study.

The two source variables and the mediating variable were latent constructs. Cueing strategy theory calls for inward indicators or productive relationships for the source variables. As directed by Van de Geer (1971, formula 13.15) factor loadings were used as path weights for the inward indicator cueing strategy variables. The "basic skills" composite was conceived as a reflective model; hence, factor score coefficients served as path weights on the outward indicators.

Notwithstanding the use of latent constructs as a device to reduce the collinearity problem, the data for the inner relationships were inherently collinear, and especially in the reciprocal effects model. Thus, the inner relationships of both the covariance and reciprocal effects models were estimated using normalization regression analysis -- a variant of ridge regression -- which uses a biased, reduced-variance estimating procedure. The method and its several advantages are discussed in the technical appendix.

5. Results

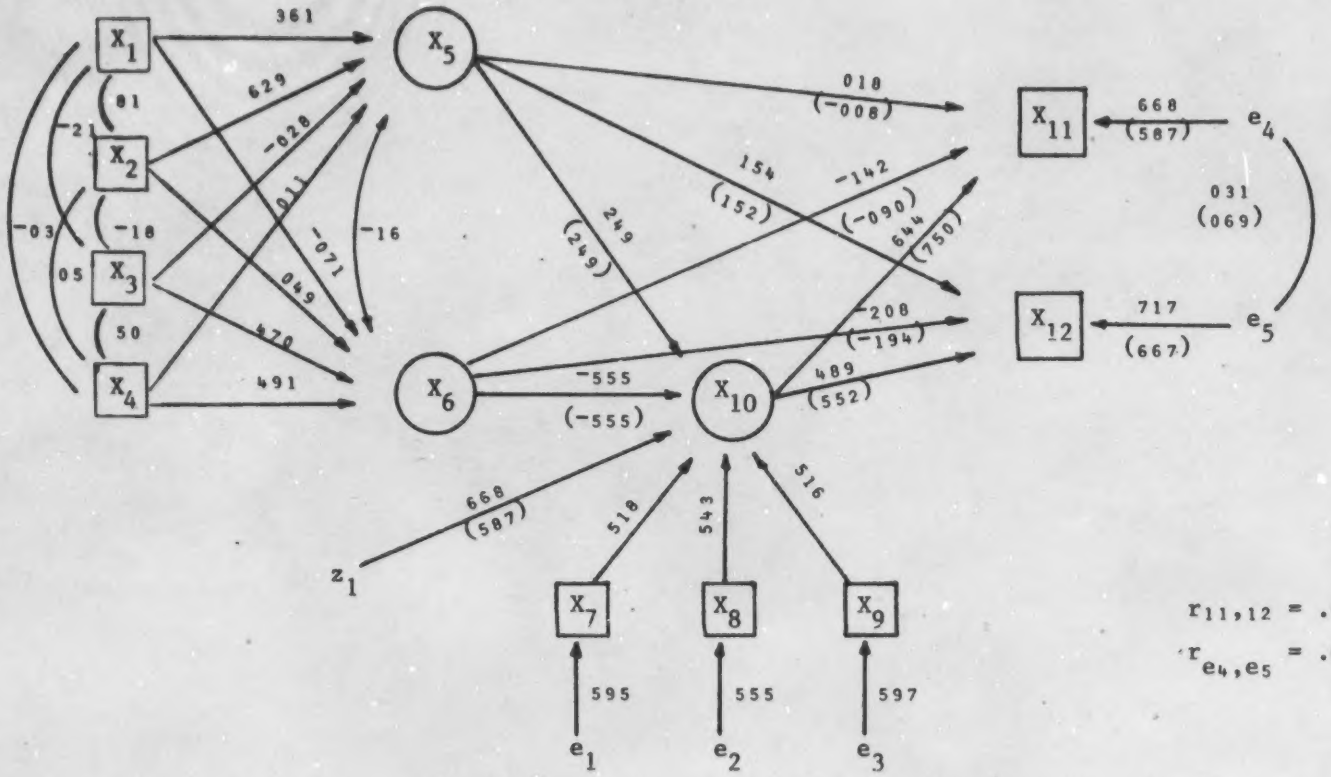
Covariance Model Findings

The structure of relationships for the covariance model is presented in Figure 1. The structural parameters of both the outer and inner relationships were estimated from the appropriate matrices of the correlation coefficients presented in Table 2. Two sets of estimates are presented in Table 3. The ordinary least squares estimates are displayed in parentheses below the normalization regression estimates in each instance. The conclusions drawn from all the analyses were based on the normalization regression estimates. See the technical appendix for reasons justifying the interpretation of the normalization regression results rather than OLS results. In Figure 1 the normalization path estimates for the inner relationships are above the paths while OLS path estimates are in parentheses below the paths.

Figure 1, Table 2 and Table 3 about here

Two Table 3 indices require explanation; namely D_{max} and k_N. D_{max} refers to the degree of collinearity. In the basic skills segment of the covariance model (Table 3) the degree of multicollinearity was negligible ($D_{max} = .012$). This is because the grapho-phonic and syntactic-semantic cueing strategies are virtually orthogonal; hence, the predictor vectors are almost completely linearly independent. Under this condition the normalization regression estimates and the OLS estimates are the same; and in fact, this is the case. Though several indicators of the severity of multicollinearity have been used by different researchers (e.g., Farrar and Glauber, 1976; Marquard and Snee, 1975; Price, 1977; White, 1978) the

Figure 1: Structure of the Covariance Model*



* Variable identifications: X_1 = proportion of miscues lacking graphic similarity, X_2 = proportion of miscues lacking phonetic similarity, X_3 = proportion of unacceptable semantic miscues, X_4= proportion of unacceptable syntactic miscues, X_5 = grapho-phonic cueing strategy deficiency, X_6 = syntactic-semantic cueing strategy deficiency, X_7 = vocabulary, X_8 = language skills, X_9 = work study skills, X_{10} = basic skills, X_{11} = reading comprehension, X_{12} = mathematics achievement. OLS estimates are below the normalization regression estimates in each instance. The latent variables are enclosed in circles; observed variables are in boxes. Decimal points omitted.

Table 2

Means, Standard Deviations and Zero-Order Correlations
for Variables in the Literacy and Numeracy Models (N = 94)

Variables		X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}	X_{12}	$\bar{X}$	SD
X_1	CS1	1.0000												19.342	9.481
X_2	CS2	.8122	1.0000											21.786	10.097
X_3	CS3	-.2101	-.1818	1.0000										35.797	18.889
X_4	CS4	-.0346	.0532	.5000	1.0000									32.538	13.047
X_5	GP	.9212	.9747	-.2250	.0158	1.0000								-.020	.940
X_6	SS	-.1966	-.1043	.9040	.8189	-.1631	1.0000							-.030	.711
X_7	VOCAB	.3151	.2175	-.5559	-.3588	.2764	-.5533	1.0000						46.904	9.282
X_8	LAN	.3289	.2778	-.5369	-.3796	.3208	-.5482	.6693	1.0000					47.349	9.094
X_9	WORK	.3033	.2748	-.4799	-.2870	.3082	-.4637	.6553	.6590	1.0000				45.627	8.056
X_{10}	SKILL	.3571	.2807	-.6008	-.3926	.3351	-.5988	.9277	.8612	.8335	1.0000			.028	1.197
X_{11}	READ	.2891	.2072	-.5542	-.3469	.2598	-.5445	.7297	.6335	.7811	.8059	1.0000		45.470	9.317
X_{12}	MATH	.3514	.3215	-.5159	-.3882	.3575	-.5379	.5705	.6131	.7167	.6972	.6154	1.0000	44.386	6.829

Variable identifications: X_1 = proportion of miscues lacking graphic similarity, X_2 = proportion of miscues lacking phonetic similarity, X_3 = proportion of unacceptable semantic miscues, X_4 = proportion of unacceptable syntactic miscues, X_5 = grapho-phonic cueing strategy deficiency, X_6 = syntactic-semantic cueing strategy deficiency, X_7 = vocabulary, X_8 = language skills, X_9 = work study skills, X_{10} = basic skills, X_{11} = reading comprehension, X_{12} = mathematics achievement.

Table 3

Path Coefficients and t-values for the OLS and Normalization Regression Estimates of the Covariation between Literacy and Numeracy*

Independent Variables	Dependent Variables					
	X_{10} SKILL		X_{11} READ		X_{12} MATH	
	β	t	β	t	β	t
X_5 GP	.249	3.116	.018	0.319	.154	2.360
	(.250)	3.107	(-.008)	-0.125	(.152)	2.052
X_6 SS	-.555	-6.959	-.142	-2.271	-.208	-2.293
	(-.560)	-6.954	(-.099)	-1.290	(-.194)	-2.229
X_{10} SKILL			.644	10.010	.489	6.684
			(.750)	9.323	(.552)	6.041
R^2	.4079		.5531		.4853	
	(.4146)		(.6556)		(.5556)	
Residual	.769		.668		.717	
	(.765)		(.587)		(.667)	
k_N	.009		.110		.110	
D_{max}	.012		.392		.392	

* The OLS regression estimates are in parentheses below the normalization regression estimates. The correlation between the residuals (e_4 and e_5) is:

$$r_{e_4,e_5} = r_{11,12\cdot5,6,10} = (R^2_{12\cdot5,6,10,11} - R^2_{12\cdot5,6,10} / 1 - R^2_{12\cdot5,6,10})^{\frac{1}{2}}$$

= .031 (normalization regression analysis result)

= .069 (OLS regression analysis result)

t-values greater than 2.0 are significant at the $p \leq .05$.

one used here was developed by Lee (1979) which gives zero for no collinearity and one for perfect collinearity. This is obtained through a simple transformation of the minimum variance inflation factor (V_{max}); given by,

$$D_{max} = \frac{2}{\pi}\tan^{-1}(V_{max} - 1)$$

and where the arc tangent is measured in radians.

The D_{max} for the second stage of the model in Table 3 was higher than that at the first stage (D_{max} = .392 versus .012) but collinearity below 0.5 is seldom serious. Nevertheless, the OLS estimates were slightly inflated; for example, $p_{11,10}$ and $p_{12,10}$ OLS estimates were too high.[8]

The normalization regression procedure is designed to reduce variance inflation in the estimates through normalizing the variance inflation factor such that it is equal to one. The value of k_N (read, "normalization k") which satisfies this condition at the second stage of the model is .110 (see Table 3). After the ridge constant (k_N=.110) has been applied to the stage two matrix the multicollinearity index may be recalculated. In the present case the recalculated D_{max} came to .061 -- a 55 percent reduction in the multicollinearity of the normalization regression matrix compared to that of the OLS matrix.

The most important covariance model finding concerned the correlation between the residuals; namely,

$r_{e_4e_5} = r_{11,12\cdot5,6,10} = (R^2_{12\cdot5,6,10,11} - R^2_{12\cdot5,6,10}/1 - R^2_{12\cdot5,6,10})^{\frac{1}{2}}$,

which came to .031 (compared to $r_{11,12}$ = .615), indicating that nearly all the covariance between reading and numeration was accounted for by the common causes -- the grapho-phonic and syntactic-semantic strategies, and basic skills. This is what the model was designed to do, but note

that it is also powerfully predictive of both reading comprehension and numeration -- accounting for 55 and 48 percent of the variance respectively. For readers interested in significance levels t-values greater than 2.0 are significant at the $p<.05$ level.

The covariance model has served its purpose well. Support for hypothesis (iii) was convincing. Nevertheless, the theory posits the existence of reciprocal relationships between reading and numeration -- a task undertaken in a respecified model in the next section.

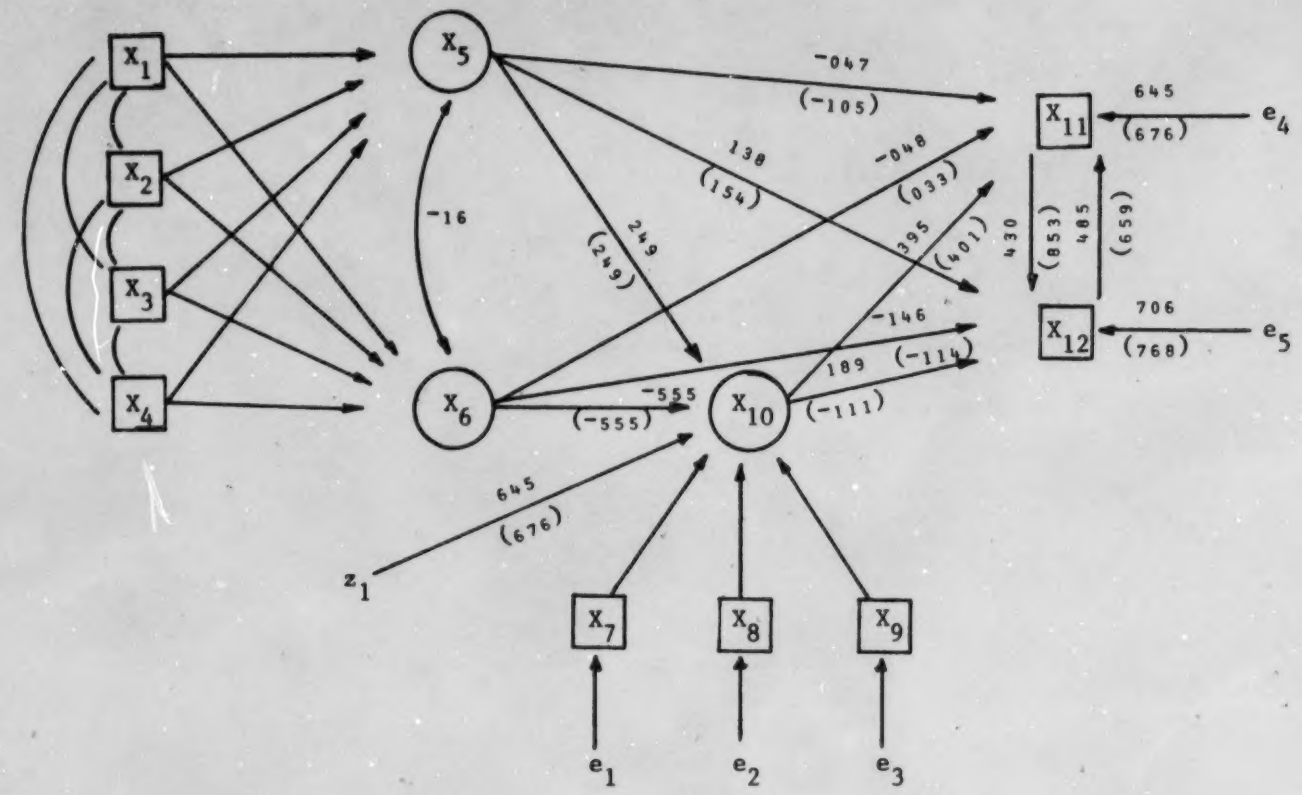
Reciprocal Effects Model Findings

The structure of the feedback model is shown in Figure 2. The estimates are presented in Table 4. Because feedback effects aggravate multicollinearity the D_{max} coefficients are high (.743 and .604) for the second and third equations; and, therefore, Table 4 results illustrate almost every dilemma associated with the multicollinearity problem. The 2SLS t-values indicate that the standard errors are unacceptably high; one of the 2SLS paths ($p_{12,10}$) has the wrong sign, and several 2SLS estimates are seriously inflated (e.g., $p_{11,12}$ and $p_{12,11}$). When the variance inflation of the estimates is reduced by the normalization procedure several hypotheses are supported.

Figure 2 and Table 4 about here

First, the basic skills mediate much of the effects of the cueing strategies on the reading and numeration outcomes. Second, the reading-numeration relationship is reciprocal, though numeration affects reading slightly more than the other way round, ceteris paribus. Note, especially,

Figure 2: Structure of the Reciprocal Effects Model*



* Variable Identifications: X_1 = proportion of miscues lacking graphic similarity, X_2 = proportion of miscues lacking phonetic similarity, X_3 = proportion of unacceptable semantic miscues, X_4 = proportion of unacceptable syntactic miscues, X_5 = grapho-phonic cueing strategy deficiency, X_6 = syntactic-semantic cueing strategy, deficiency, X_7 = vocabulary, X_8 = language skills, X_9 = work study skills, X_{10} = basic skills, X_{11} = reading comprehension, X_{12} = mathematics achievement. The structure of the outer relationships is the same as that in Figure 1. The 2SLS estimates are in parentheses below the 2SNR estimates. Decimal points have been omitted.

Table 4

Reciprocal Effects Model Results: Path Coefficients and t-ratios from Two-Stage Normalization Regression (2SNR) and Two-Stage Least Squares (2SLS) Analyses*

	Dependent Variables					
Independent	X_{10} SKILL		X_{11} READ		X_{12} MATH	
Variables	β	t	β	t	β	t
X_5 GP	.249 (.250)	3.116 3.107	-.047 (-.105)	-0.696 -1.096	.138 (.154)	1.918 1.590
X_6 SS	-.555 (-.560)	-6.959 -6.954	-.048 (.033)	-0.647 0.286	-.146 (-.114)	-1.839 -0.950
X_{10} SKILL			.395 (.401)	5.271 2.091	.189 (-.111)	2.522 -0.344
X_{11} READ					.430 (.853)	4.877 2.145
X_{12} MATH			.485 (.659)	5.016 2.164		
R^2	.4079 (.4146		.5839 (.5434)		.5014 (.4094)	
Residual	.769 (.765)		.645 (.676)		.706 (.768)	
k_N	.009		.152		.149	
D_{max}	.012		.743		.604	

* The 2SLS estimates are in parentheses below the 2SNR estimates.

t-ratios greater than 2.0 are significant at the $p \leq .05$

that if the 2SIS results had been interpreted the opposite conclusion -- the one supportive of the traditional explanation of the literacy-numeracy relationship -- may have been drawn. The fine grain relationships in the model can be seen more effectively by examining total indirect effects and total causal effects.

Indirect Effects Analysis

The pathways of structural equation models can be traced from effects back to causes. It is easy to show that X_{10} (SKILL) directly affects X_{11} (READ) via $p_{11,10}$. It is less easy to "see" how (say) X_4 -- the proportion of unacceptable syntactic miscues -- affects X_{11} (READ), or whether X_4 has as powerful an impact on X_{12} (MATH) as X_{11} (READ). The causal impact of X_4 on X_{11} or X_{12} can be estimated if their correlations are partitioned into causal and noncausal components. The noncausal component is that portion of the association due to common causes. The causal component -- the part of interest -- consists of the direct effects plus the indirect effects of X_4 on X_{11}. In the $r_{4,11}$ example there is no direct path, thus the total causal effect of X_4 on X_{11} is the sum of all the indirect effects via intervening variables; namely, $p_{64}p_{11,6} + p_{54}p_{11,5} + p_{64}p_{10,6}p_{11,10} + p_{54}p_{10,5}p_{11,10} + p_{64}p_{10,6}p_{12,10}p_{11,12} + p_{54}p_{10,5}p_{12,10}p_{11,12}$. In Table 5 the total indirect effects (TIEs) and total causal effects (TCEs) for all predictor variables are presented. The TCE of X_4 on X_{11} is -.155; about the same as that of X_4 on X_{12} (-.166). In other words, the higher the proportion of unacceptable syntactic miscues the lower the performances in both reading and numeration.

The Table 5 findings support hypothesis (ii); namely, that numeration

and reading outcomes are more responsive to syntactic and semantic strategies than to the graphic and phonic. For example, in the case of reading the TCEs of the proportion of miscues lacking graphic similarity and phonetic similarity are a negligible .040 and .031 respectively, while the TCEs of the proportion of unacceptable semantic and syntactic miscues are much greater in magnitude; namely, -.152 and -.155.

If the TCEs of the predictors of reading are rank ordered it can be shown that the basic skills and numeration have the greatest impact (.487 and .485 respectively). The TCEs of X_6, X_4 and X_3 -- the set of syntactic-semantic strategies -- are also highly significant. Reading has the most powerful TCE on numeration (.430) while the impact of basic skills (.359) and syntactic-semantic cueing strategy (-.345) is about equal. But how come the TCEs of the grapho-phonic cueing strategy variable X_5 on X_{10} (SKILL) and X_{12} (MATH) are positive? In a literal sense this means that the greater the grapho-phonic cueing strategy deficiency the better the basic skills and numeration performances. This question is addressed in the conclusion.

Table 5 about here

Table 5

Total Causal Effects of Predictor Variables in the Reciprocal Effects Model (2SNR Analysis Results)

Independent Variables	Depen. Variables	TIE through Intervening variables	Direct Effects	Total Causal Effects (TCEs)
X_1 CS1	X_5	--	.361	.361
	X_6	--	-.071	-.071
	X_{10}	.129	--	.129
	X_{11}	.049	--	.049
	X_{12}	.107	--	.107
X_2 CS2	X_5	--	.629	.629
	X_6	--	.049	.049
	X_{10}	.129	--	.129
	X_{11}	.031	--	.031
	X_{12}	.111	--	.111
X_3 CS3	X_5	--	-.028	-.028
	X_6	--	.470	.470
	X_{10}	-.268	--	-.268
	X_{11}	-.152	--	-.152
	X_{12}	-.169	--	-.169
X_4 CS4	X_5	--	.011	.011
	X_6	--	.491	.491
	X_{10}	-.270	--	-.270
	X_{11}	-.155	--	-.155
	X_{12}	-.166	--	-.166

Table 5 (cont'd.)

Indep. Variables	Depen. Variables	TIE through Intervening variables	Direct Effects	Total Causal Effects (TCEs)
X_5 GP	X_{10}	--	.249	.249
	X_{11}	.121	-.047	.074
	X_{12}	.089	.138	.227
X_6 SS	X_{10}	--	-.555	-.555
	X_{11}	-.270	-.048	-.318
	X_{12}	-.199	-.146	-.345
X_{10} SKILL	X_{11}	.092	.395	.487
	X_{12}	.170	.189	.359
X_{11} READ	X_{12}	--	.430	.430
X_{12} MATH	X_{11}	--	.485	.485

Elasticities Analysis

(1) What will happen to reading performance if mathematics performance changes? or (2) What will happen to the basic skills performances on the basis of a change in a child's syntactic-semantic cueing cueing strategy? Such questions are seldom posed by educational researchers, though they are commonplace in economics dating back to Alfred Marshall's (1890/1961) research on the elasticity of demand.

In general, elasticity refers to a measure of the degree of responsiveness of a dependent variable to changes in an independent variable, ceteris paribus. Consider the mathematics elasticity of reading (the first of the above questions) which refers to the responsiveness of reading as a dependent variable to changes in mathematics performance as the independent variable. From Table 6b it can be seen to be .6465; and we can say, therefore, that about the means of the variables (that is, for the typical grade four pupil) a ten percent increase in mathematics achievement causes a six percent increase in reading performance. With reference to the second question -- namely, the syntactic-semantic elasticity of basic skills -- it can be seen (Table 6a), that the school dependent skills are of unit elasticity; that is, there is a one-to-one correspondence (E_j = 1.005) such that a ten percent decline in the syntactic-semantic cueing strategy deficiency causes a ten percent improvement in basic skills performance.

Table 6 about here

The utility of an elasticities analysis stems from the fact that the elasticities are unit free; that is, their values are independent of the metric of the variables. Large elasticities (those usually greater than 1.0) imply that the dependent variable is highly responsive to change in the

Table 6

Elasticities Analysis Results for Two-Stage Normalization Regression Analysis of the Reciprocal Effects Model*

Table 6a. Normalization Regression Equation No. 1: $X_{10}(SKILL) = f(X_5, X_6)$

Indep. Variables	$\bar{X}_j$	Coefficients (β_j)	Elasticities (E_j)
X_5 GP	-.020	.3090	-.2245
X_6 SS	-.030	-.9333	1.0005

$\bar{Y} = \bar{X}_{10}(SKILL) = .208$; $k_N = .009$

Table 6b. 2SNR Equation No. 2: $X_{11}(READ) = f(X_5, X_6, X_{10}, \hat{X}_{12})$

Indep. Variables	$\bar{X}_j$	β_j	E_j
X_5	-.020	-.4628	.0002
X_6	-.030	-.6228	.0004
X_{10}	.028	3.0717	.0019
$\hat{X}_{12}$	44.386	.6623	.6465

$\bar{Y} = \bar{X}_{11}(READ) = 45.470$; $k_N = .152$

Table 6c. 2SNR Equation No. 3: $X_{12}(MATH) = f(X_5, X_6, X_{10}, \hat{X}_{11})$

Indep. Variables	$\bar{X}_j$	β_j	E_j
X_5 GP	-.020	1.0041	-.0005
X_6 SS	-.030	-1.4072	.0009
X_{10} (SKILL)	.028	1.0766	.0007
$\hat{X}_{11}$ (READ)	45.470	.3149	.3226

* Where $E_j = \hat{\beta}_j(\bar{X}_j/\bar{Y})$. Interpretation: If $E_j = 1.0005$ (Table 6a) we say that about the mean of the variables a one percent increase in X_6(SS) will lead to a one per cent increase in $Y(X_{10}$ - SKILL)

independent variable.

Since on the basis of earlier results (e.g., Figure 2) we were able to show that the basic skill variable mediates the effects of cueing strategies on reading and numeration; and since the mediating effect of the basic skill variable was confirmed by the TCEs in Table 5, it is readily assumed that a viable way of improving reading comprehension and numeration performances would be to develop the students' basic skills. The conclusion is all the more acceptable because it reinforces current pedagogical theory and practice in the middle grades.

Such a conclusion, however, would be premature. It is not supported by the basic skill elasticities reported in Tables 6b and 6c. In Table 6b, for example, one can see that the only large E_j is for MATH; that is, reading comprehension is responsive to changes in numeration but to nothing else. Thus, for the typical grade four pupil reading is responsive to changes in mathematics performance but not to changes in basic skills. According to the results, a one hundred percent improvement in basic skills would probably only lead to a two percent improvement in reading. Similarly, numeration performance (Table 6c) is responsive to changes in reading but not to changes in other predictors.[9]

In terms of the conventional pedagogical wisdom the elasticities results are disappointing. The efforts of teachers in terms of teaching the basic skills may have little effect in terms of children's reading and numeration performances. This is not to say that teaching the basic skills may not have desirable consequences for anything else, or that their effects may not be lagged. There is a sense in which the basic skills -- the development of meaning vocabulary, language usage skills and work study skills -- are intrinsically worth while.

6. Conclusion

There was substantial support for three of the four hypotheses. First, the covariance model results (Table 3) demonstrated that most of the covariation between literacy and numeracy was attributable to the grapho-phonic and syntactic-semantic cueing strategies, along with the basic skills. Second, the indirect effects analyses (Table 5) supported the hypothesis that the syntactic-semantic cueing strategy for grade four students would have greater impact on literacy and numeracy than the grapho-phonic strategy. Third, the reading-numeration model was reciprocal (Figure 2) such that both outcomes were approximately equal in their feedback effects on the other. And, paradoxically, though the basic skills mediated the effects of the cueing strategies, this did not mean that reading and numeration outcomes were responsive to these predictors which proved to be inelastic (Table 6). Fourth, since the theory was based on the notion that analysis-by-synthesis decoding was common to speech recognition, arithmetical problem solving, and reading comprehension; and since the decoding of speech is prior to the other two, there was *prima facie* support for the thesis that the origins of both literacy and numeracy are to be found in the logic of speech perception. Further, the reciprocal effects model suggests that reading and numeration are co-extensive developments which are consequential to the acquisition of the analysis-by-synthesis logic of speech recognition. Collectively, these findings strongly support the new theory; hence, promote rejection of the "primacy of reading" model.

Support for the first hypothesis, however, was equivocal; namely, that the basic skills mediate the effects of intuitive linguistic rule-following or cueing strategies of reading and numeration. Support for the hypothesis

was forthcoming in three instances: (i) the covariance model results (Figure 1), (ii) the reciprocal effects model results (Figure 2), and (iii) the indirect effects analysis (Table 5). But these findings were not confirmed by the elasticities analysis. We do not know why, and we caution the reader to exercise a healthy scepticism in so far as the elaticities results are concerned. Nevertheless, the elasticities could only be interpreted to show that about their means the reading and numeration variables were only responsive to each other; not to the source variables, nor to the basic skills.

In retrospect we tentatively provide the following interpretation regarding hypothesis one. On the basis of the models estimated in the study we can only infer that the reader or problem solver is on the inside of the logic of analysis-by-synthesis decoding by noting the pupil's proficiency in employing the cueing strategies. The processes themselves -- hypothesizing, predicting, inferencing and others -- are not amenable to direct observation. In this sense, the cueing strategies are proxies or surrogate variables which stand for these processes -- but which ones, and to what effect we do not know. It could well be that the proxies in the formulated model are inadequate. It is what they stand for that is important, but as yet our research skills are not equal to the task -- which in and of itself may not be surprising given the youthfulness of the incremental model building tradition in educational research. This is not to say we are back to where we started. The research has still to be replicated, extended, and elaborated on.

We need more experimental research in cognitive processing. The first principle of the research tested in this study was that the logical roots of numeration and reading performance lie in the analysis-by-synthesis model developed by speech recognition researchers. One viable research route, then,

might be to to try to further disaggregate the cueing strategies -- say, by examining the extent to which they are complemented in their effects by the classes of logical connectives and information units that children use in their retelling discourse. Fagan's (1978) recent research is at the cutting edge of this tradition, though as yet the formal models integrating miscue analysis and discourse analysis have not been estimated. Aural recognition anlysis, though undeveloped, would also seem a viable route for the reading researcher to take because aural recognition is developmentally prior to literacy and numeracy.

A final anomaly. How come grapho-phonic deficiency is a good predictor of both basic skills and numeration? See Table 5, where it is shown that the TCEs for variable X_5 on X_{10} and X_{12} are positive (.249 and .227 respectively). It seems that those grade four pupils who have successfully made the transition from learning to read to reading to learn take grapho-phonic skills for granted. These meaning seekers overlook grapho-phonic miscues (that is, regard them as expendable) if they do not detract from the flow of meaning. They place comprehension before sounding out words -- and the two are not particularly complementary -- even if it is at the expense of grapho-phonic perfection. Obversely, grapho-phonic accuracy at this level, as Kenneth Goodman was the first to point out, is all too often at the expense of comprehension.

Technical Appendix:

On the Method of Normalization Regression Analysis

There is a growing awareness of the severity of the multicollinearity problem by survey analysts in the structural equation tradition. Thus, it is noted that in the presence of highly intercorrelated predictor variables, ordinary least squares (OLS) estimates are likely to be too large in magnitude, too sensitive to sampling error and specification error, and possibly of the wrong sign (see, for example, Hoerl and Kennard, 1970, or Lee, 1979).

Consider the standardized regression model:

$$y = \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 \ldots + \beta_p X_p + \varepsilon,$$

where, (1) the β's are the true regression coefficients;

(2) the X's are explanatory variables which may be

(i) fixed or random,

(ii) orthogonal or nonorthogonal;

(3) ε is the random error in y;

(4) y is a random variable which satisfies the multinormal distribution; and where

(5) the model could be either true or designed.

When the model is orthogonal, for example,

$$y = \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \hat{\beta}_3 X_3 + \ldots + \hat{\beta}_p X_p + u,$$

it will have the following features.

(1) All the $\hat{\beta}$'s will have the same sampling variance; namely, $\text{Var}(\hat{\beta}_1) = \hat{\sigma}^2 = \frac{\Sigma u^2 i}{n-p}$. (Note that this variance is that obtained by repeated sampling. When a different sample is used we will obtain different values for the $\hat{\beta}$'s. These $\hat{\beta}$'s will be normally distributed with variance $\hat{\sigma}^2$.)

(2) The $\hat{\beta}$'s are not correlated.

(3) The $\hat{\beta}$'s are partial regression coefficients.

(4) The $\hat{\beta}$'s do not depend on the number of variables p.

(5) The residual u is not necessarily equal to ε.

But in the real world multicollinearity is always a matter of degree, not of kind; thus, in practice any $p > 1$ multiple regression problem will be nonorthogonal with the following characteristics.

(1) Each $\hat{\beta}$ has a different sampling variance.

(2) The sampling variance of $\hat{\beta}_i$ ($i = 1,2, \ldots, p$) will be inflated by a variance inflation factor (VIF_i) due to the presence of collinearity such that, $\text{Var}(\hat{\beta}_i) = \hat{\sigma}^2 \, VIF_i$.

(3) The variance inflation (VIF) can approach infinity when the data set has a high degree of multicollinearity.

(4) Because of high sampling variance the $\hat{\beta}$'s
   (i) are usually over-estimated in terms of magnitude;
   (ii) can be of the wrong sign;
   (iii) may prove to be unreliable; and
   (iv) tend to be particularly sensitive to sampling error and specification error.

(5) The $\hat{\beta}$'s are correlated so that, in general, if r_{j1} is high, the $\hat{\beta}_j$ and $\hat{\beta}_1$ are negatively correlated; that is, if $\hat{\beta}_j$ is over-estimated, $\hat{\beta}_1$ will be under-estimated.

(6) The $\hat{\beta}$'s are unbiased estimates only if the model is a true model. Empirically, therefore, the $\hat{\beta}$'s are biased estimates.

(7) In the nonorthogonal model the R^2 becomes progressively less meaningful as a function of the degree of multicollinearity.

(8) The forecasting variance of the nonorthogonal model is:

$$\hat{\sigma}_f^2 = \frac{(1 - R^2)\Sigma y_i^2}{n - p}\left[1 + \Sigma \frac{x_i^{*2}}{\lambda_i}\right]$$

In contrast the ridge regression model is:

$$y = \hat{\beta}_1^* X_1 + \hat{\beta}_2^* X_2 + \hat{\beta}_3^* X_3 + \ldots + \hat{\beta}_p^* X_p + u$$

(1) The estimates $\hat{\beta}^*$'s are negatively biased even if the model is a true one.

(2) The ridge regression model which uses the variance normalization criterion constrains the average VIF to unity.

(3) Each $\hat{\beta}^*$ in the variance normalization model has approximately the same VIF ($\simeq 1$).

(4) The larger the VIF for an estimate the greater the reduction in the VIF through application of the ridge regression procedure.

(5) The effects of ridge regression on the $\hat{\beta}^*$'s, depends on both the variance inflation factor (VIF) and the covariance inflation factor (CIF).

(6) The reduction of the total variance is highest for low k (the biasing parameter); this can be seen from,

$$\Sigma \text{Var}\ (\hat{\beta}_i^*) = \sigma^2 \Sigma \frac{\lambda_i}{(\lambda_i + k)^2}$$

(7) The squared bias (Bias^2) cannot be calculated, but the rate of increase in bias is higher for large k and low for low k; this is clearly indicated by,

$$\text{Bias}^2(\hat{\beta}^*) = k^2 \Sigma \frac{\sigma_i}{(\lambda_i + k)^2}$$

(8) The total variance reduces to zero when $k \to \infty$.

(9) The model has greater forecasting power than the OLS regression models (for proof see Lee, 1979):

$$\hat{\sigma}_f^{*2} = \sigma^2 \left[1 + \Sigma \frac{\lambda_i x_i^{*2}}{(\lambda_i + k)^2} \right] + \text{Bias}^2\ (\hat{\beta}^*)$$

These characteristics stem from the work of Hoerl and Kennard (1970) who found that in order to reduce the total sampling variance of the estimates, the diagonal elements of the correlation matrix had to be augmented by a constant term k (called the biasing parameter). This was obtained in the Hoerl and Kennard model by minimizing the total mean squared error (MSE) of the estimates as follows:

$$\text{MSE} = E\left[\Sigma(\hat{\beta}_i^* - \beta_i)^2\right]$$

$$= \Sigma \text{Var}(\hat{\beta}_i^*) + \text{Bias}^2\ (\hat{\beta}^*) + \text{minimum}$$

where $\Sigma \text{Var}(\hat{\beta}_i^*)$ = the total variance of the estimates

= the sum of the squares of the random errors of the estimates.

It is argued, however, that the minimum MSE is not a proper criterion for ridge regression for several reasons. First, it is inadmissible because k depends on unknown population parameters β and σ^2. Second, k is stochastic while theoretically it should be nonstochastic. Third, it does not satisfy the boundary condition; that is, for orthogonal data ridge regression produces different estimates than does OLS regression.

Thus, an alternative, called the Variance Normalization Criterion, is proposed in which the biasing parameter k (called k_N) "normalizes"

the total variance of the estimates as follows:

$$\Sigma \mathrm{Var}(\hat{\beta}_i^*) = p\sigma^2 \quad \text{...in the ideal case, an orthogonal data set,}$$

$$\text{or, } \frac{1}{p} \Sigma_i \frac{\lambda_i}{(\lambda_i + k)^2} = 1$$

The advantages of the new criterion are as follows: (i) k_N can be calculated accurately; (ii) $0 \leq k \leq .25$; (iii) it is nonstochastic; that is, it is not dependent on y, β, or σ^2; (iv) it satisfies the boundary condition; and (v) maximizes the reduction of the harmful effects of muticollinearity. Thus, it corrects several inherent limitations of the earlier approach based on the minimum MSE criterion; and, therefore, in practice performs more consistently. For several applications see Bulcock (ed. 1979; especially Lee and Luk, 1979). Application of the new criterion in single equation estimation procedures is called "normalization regression analysis".

Footnotes

1. A paper stemming from this project was presented by Beebe at the Annual Meetings of the Canadian Society for the Study of Education, Saskatoon, June, 1979. The research on the methodological procedures was supported by Grant No. S86-786 awarded to the first author by the Social Sciences and Humanities Research Council. W. F. Lee, Memorial University of Newfoundland, was the Research Associate on the project, and W. S. Luk, Simon Fraser University, was the Research Consultant. The authors gratefully acknowledge their contribution and point out that without their assistance the estimating techniques described in the technical appendix could not have been developed.

2. To avoid unnecessary confusion, it is pointed out that in the interests of style, the terms "literacy" and "reading comprehension" are used interchangeably throughout the study; as are the terms "numeracy", "numeration", "arithmetical problem solving" and "verbal arithmetic problem solving"; and the terms "speech perception" and "speech recognition."

3. Hollander (1978) in her review of the research related to verbal arithmetic problem solving identified twelve studies in which either general reading ability or specific reading skills were used as factors predictive of arithmetic problem solving. All but one study found gains in reading associated with concomitant gains in arithmetic performance. The usual conclusion on the basis of this kind of evidence is that reading deficiencies hinder abstract thinking as represented by computational skill and verbal arithmetic problem solving. Hollander, however, did not report the results of research on the most verbally deprived of children -- the congenitally deaf. A review of these studies by Vernon (1968) shows that while deaf children score well below normal children on verbal tests "the research of the last fifty years ... indicates that when there are no complicating multiple handicaps, the deaf

and hard-of-hearing function at approximately the same IQ level on performance tests as do the hearing." (1968, p. 9).

4. Hauser (1972, p. 78) estimated the reciprocal effects between reading and mathematics achievement for a large sample of Tennessee high school seniors. He found that reading had a powerful impact on mathematics, but that the effects of mathematics achievement on reading were modest. We would regard his model, however, as seriously missecified. Further, prudence dictates that this finding not be over-generalized to apply to elementary school children.

5. The sceptic could take issue with these results on at least two grounds. In the first place the findings could be a function of the selection of an arbitrary criterion of mastery in reading and numeration. Though it is conceded that cutting score criteria will not eliminate the need for researcher judgment, the Australian researchers did not select the cutting scores arbitrarily either. (See Keeves and Bourke, 1977; Morgan, 1977). In the second place there is the cultural styles hypothesis to consider. Effectively, the hypothesis holds that the differences in knowledge and problem solving abilities as between different societies and cultures represent fundamental differences in thought processes (see Berry, 1976; Berry and Dasen, 1974; Cole and Scribner, 1974; Lloyd, 1972; and Luria, 1976, for major contributions to this literature). According to this cultural relativity thesis there may be little reason to suppose that Australian findings would be replicable elsewhere; say, Canada. Such a position, however, is probably far-fetched. Cultural differences, even between socio-historically different nations need not be that great as to support a logically different styles of thinking hypothesis. Cross-national research in school achievement between Western nations (Western and Eastern European) supports a structuralist perspective; namely, that

the principles underlying human diversity suggest few fundamental differences in human thinking across socio-historically different societies. (For example, see Bulcock and Lee, 1980; Bulcock et al., 1979; Clifton, 1978; and Munck, 1979). For major contributions to the structuralist perspective see Chomski (1968), Laughlin and d'Aquili (1974), Levi-Strauss (1966), and Piaget (1973).

6. Small children are provided with a great deal of guidance by their private tutors (mothers). A child's prelinguistic communication takes place in a setting where the tutor provides the tell-tale clues for the child's hypothesis testing behaviour. Bruner's (1978) research confirms that children in these intimate settings are able to infer and generate the rules governing the structure of discourse. Siegel (1978) has questioned whether day care centers provide optimal environments for language development.

7. Richard Noonan's research using Partial Least Squares (PLS) procedures developed by Wold provides outstanding examples of the use of latent variable modelling in education. (For example, see Noonan and Wold, 1977, 1978).

8. A word of explanation. Ill-conditioned or highly collinear data sets have linear vectors which depart from the condition of independency or orthogonality. In these circumstances OLS regression estimates will be inflated. This is because the data matrices will have high variance and covariance; hence, the estimates will be highly correlated. Because of the independency of the estimates, if one is erroneous, the other will be; for example, when two predictors are highly positively correlated the estimated coefficients will be highly negatively correlated. This means that if one coefficient is over-estimated the other will be under-estimated. It is this high correlation between the estimates which accounts for the excessively large sum of squares $(\hat{\beta}'\hat{\beta})$ in the OLS solutions and why it is not uncommon for the estimates to have the wrong sign. Thus, the OLS estimate (see Figure 1 or Table 3) for

$p_{11,10}$ is over-estimated, while $p_{11,6}$ is under-estimated. This common problem in educational research is dealt with more authoritatively in several sources (Althauser, 1971; Blalock, 1963, 1964; Christ, 1966; Gordon, 1968; Hoerl and Kennard, 1970; Johnston, 1972; Kasarda and Shih, 1977; Kmenta, 1971).

9. One could inquire as to the elasticities of the predictors in a recursive reading model; that is, one with variables X_5, X_6, and X_{10} as predictors. They are: -.0001, .0012, and .0031 respectively. For a model of arithmetic problem solving the news is little better. The elasticities for X_5, X_6, and X_{10} are: -.0005, .0014, and .0017.

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